



education

MPUMALANGA PROVINCE
REPUBLIC OF SOUTH AFRICA

**NATIONAL
SENIOR CERTIFICATE**

GRADE 12

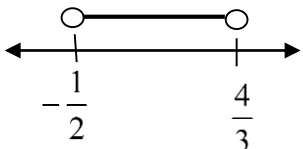
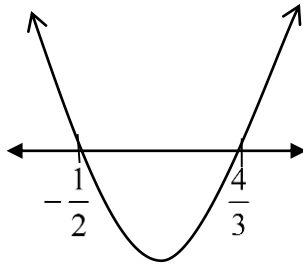
MATHEMATICS PAPER 1

SEPTEMBER 2023

MARKING GUIDELINES

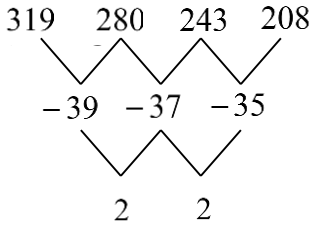
MARKS: 150

This marking guideline consists of 13 pages.

QUESTION 1 / VRAAG 1		
1.1.1	$(3-x)(2-x) = 0$ $x = 3 \text{ or } x = 2$	✓ ✓ answers (2)
1.1.2	$2x^2 + 7x = 2$ $2x^2 + 7x - 2 = 0$ $x = \frac{-(7) \pm \sqrt{7^2 - 4(2)(-2)}}{2(2)}$ $x = \frac{-7 \pm \sqrt{65}}{4}$ $x = 0,27 \text{ or } x = -3,77$ Penalise 1 mark for incorrect rounding	✓ standard form ✓ substitution into correct formula ✓ ✓ answers (4)
1.1.3	$4 + 5x > 6x^2$ $6x^2 - 5x - 4 < 0$ $(3x - 4)(2x + 1) < 0$  $-\frac{1}{2} < x < \frac{4}{3}$ OR $x \in \left(-\frac{1}{2}; \frac{4}{3}\right)$ 	✓ factors ✓ critical values ✓ ✓ answer (4)
1.1.4	$9^x + 9 = 10 \cdot 3^x$ $3^{2x} - 10 \cdot 3^x + 9 = 0$ $(3^x - 9)(3^x - 1) = 0$ $3^x = 3^2 \text{ or } 3^x = 3^0$ $x = 2 \text{ or } x = 0$	✓ 3^{2x} ✓ factors ✓ exponential law ✓ answer (4)

1.2	$y - 2x = -1$ $y = 2x - 1 \dots (1)$ $y^2 - 3xy = -2 \dots (2)$ $(2x - 1)^2 - 3x(2x - 1) = -2$ $4x^2 - 4x + 1 - 6x^2 + 3x = -2$ $-2x^2 - x + 3 = 0$ $2x^2 + x - 3 = 0$ $(2x + 3)(x - 1) = 0$ $x = -\frac{3}{2} \text{ or } x = 1$ $y = 2\left(-\frac{3}{2}\right) - 1 \text{ or } y = 2(1) - 1$ $y = -4 \text{ or } y = 1$ OR $y - 2x = -1$ $x = \frac{y + 1}{2} \dots (1)$ $y^2 - 3xy = -2 \dots (2)$ $y^2 - 3\left(\frac{y + 1}{2}\right)y = -2$ $2y^2 - 3y^2 - 3y + 4 = 0$ $-y^2 - 3y + 4 = 0$ $y^2 + 3y - 4 = 0$ $(y + 4)(y - 1) = 0$ $y = -4 \text{ or } y = 1$ $x = -\frac{3}{2} \text{ or } x = 1$	✓ equation 1 ✓ substitution ✓ standard form ✓ factors ✓ x-values ✓ y-values (6) ✓ equation 1 ✓ substitution ✓ standard form ✓ factors ✓ y-values ✓ x-values (6)
1.3	Multiply every term by LCD $4xy$ $4x^2 + 4y^2 = 17xy$ $4x^2 - 17xy + 4y^2 = 0$ $(4x - y)(x - 4y) = 0$ $4x - y = 0$ $\frac{x}{y} = \frac{1}{4}$ $x - 4y = 0$ $\frac{x}{y} = 4$	✓ multiply by LCD ✓ standard form ✓ factors ✓ answer ✓ answer (5)

	OR $\frac{x}{y} + \frac{y}{x} = \frac{17}{4}$ $\frac{x}{y} + \frac{1}{\frac{x}{y}} = \frac{17}{4}$ $\frac{x^2}{y^2} - \frac{17x}{4y} + 1 = 0$ $\frac{x}{y} = \frac{\frac{17}{4} \pm \sqrt{\left(\frac{17}{4}\right)^2 - 4(1)(1)}}{2(1)}$ $= \frac{\frac{17}{4} \pm \sqrt{\frac{225}{16}}}{2}$ $\frac{x}{y} = 4 \text{ or } \frac{x}{y} = \frac{1}{4}$	✓ fraction ✓ standard form ✓ substitution into correct formula ✓ simplification ✓ answers (5)
		[25]

QUESTION 2 / VRAAG 2		
2.1	319 ; 280 ; 243 ; 208 ; 175 ; 144 ;	
2.1.1	 $2a = 2 \quad 3a + b = -39 \quad a + b + c = 319$ $a = 1 \quad 3(1) + b = -39 \quad 1 - 42 + c = 319$ $b = -42 \quad c = 360$ $T_n = n^2 - 42n + 360$	✓ First differences ✓ $2a = 2$ ✓ $3(1) + b = -39$

		✓ $1 - 42 + c = 319$ (4)
2.1.2	$n^2 - 42n + 360 = 0$ $(n - 12)(n - 30) = 0$ $n = 12$ or $n = 30$	✓ $T_n = 0$ ✓ factors ✓ answer (3)
2.1.3	$2n - 42 = 0$ OR $n = \frac{-(-42)}{2(1)}$ $n = 21$ $n = 21$ T_{21} is lowest	✓ derivative = 0 ✓ answer (2) OR ✓ substitution into $x = \frac{-b}{2a}$ ✓ answer (2)
2.2	$3t ; 4t - 1 ; 23$	
2.2.1	$3t ; 4t - 1 ; 23$ $4t - 1 - 3t = 23 - (4t - 1)$ $4t - 1 - 3t = 23 - 4t + 1$ $5t = 25$ $t = 5$	✓ subtraction ✓ simplification (2)
2.2.2	$15 ; 19 ; 23$ $a = 15$ $d = 4$ $S_{50} = \frac{50}{2}[2(15) + (49)4]$ $= 5650$	✓ $a = 15$ ✓ value of d ✓ substitution into correct formula ✓ answer (4)
		[15]

QUESTION 3

3.1	$x = 4 + 12 + 36 + \dots$ to 15 terms	
3.1.1	$T_n = a.r^{n-1}$ $T_n = 4.3^{n-1}$ $\sum_{n=1}^{15} (4.3^{n-1})$	✓ $T_n = 4.3^{n-1}$ ✓ answer (2)
3.1.2	$S_n = \frac{a(r^n - 1)}{r - 1}$ $S_n = \frac{4(3^{15} - 1)}{3 - 1}$ $= 28697812$	✓ substitution into the correct formula ✓ answer (2)
3.1.3	$r = 3$ and not between -1 and 1 $\therefore$ not converging	✓ $r = 3$ not in $-1 < r < 1$ ✓ answer (2)
3.2.1	$S_\infty = \frac{a}{1 - r} = 13,5$ $a = 13,5(1 - r)$ $S_\infty = \frac{ar^2}{1 - r} = 1,5$ $ar^2 = 1,5(1 - r)$ $a = \frac{1,5(1 - r)}{r^2}$ $13,5(1 - r) = \frac{1,5(1 - r)}{r^2}$ $r^2 = \frac{1,5}{13,5}$ $= \frac{1}{9}$ $r = \pm \frac{1}{3}$ But $r > 0$ $\therefore r = \frac{1}{3}$	✓ substitution into correct formula ✓ $a = 13,5(1 - r)$ ✓ equating ✓ $r = \frac{1}{3}$ (4)

3.2.2	$\frac{a}{1 - \frac{1}{3}} = 13,5$ $a = 9$ $9 + 3 + 1 + \dots$	✓ value of a ✓ answer (2)
		[12]
QUESTION 4		
4.1	$0 = \frac{a}{-2+1} + 2$ $a = 2$	✓ substitution ✓ answer (2)
4.2	$\text{Average gradient} = \frac{h(-2) - h(-4)}{-2 - (-4)}$ $= \frac{\frac{2}{-2+1} + 2 - \left(\frac{2}{-4+1} + 2 \right)}{2}$ $= -\frac{2}{3}$	✓ $h(-4)$ and $h(-2)$ ✓ substitution ✓ answer (3)
4.3	$-1 < x < 0$	✓ critical values ✓ answers (2)
4.4	$y = x + 1 + 2$ $y = x + 3$ OR $y = x + c$ $2 = -1 + c$ $c = 3$ $y = x + 3$	✓ gradient ✓ answer (2) OR ✓ value of c ✓ answer (2)
4.5	$g(x) = \frac{-2}{x+1} - 2$ $x = -1$ $y = -2$	✓ horizontal asymptote ✓ vertical asymptotes (2)
		[11]

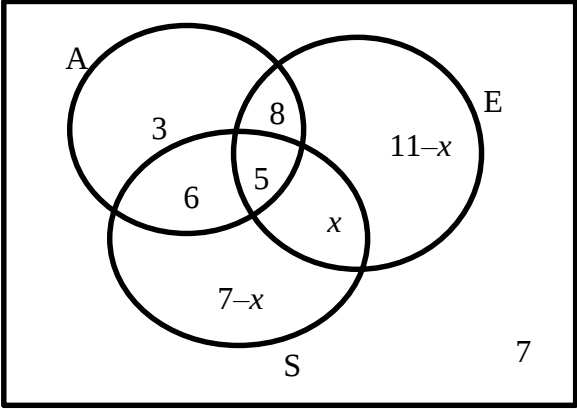
QUESTION 5		
	$h(x) = q^x \quad f(x) = -\frac{1}{2}x^2 + \frac{1}{2}x + k \quad g(x) = mx + c$	
5.1	$h(x) = q^0$ $= 1 \quad D(0; 1)$	✓ answer (1)
5.2	$k = 1$	✓ answer (1)
5.3	$f(x) = -\frac{1}{2}x^2 + \frac{1}{2}x + 1$ $x^2 - x - 2 = 0$ $(x - 2)(x + 1) = 0$ $x = 2 \text{ or } x = -1$ BC = 3units	✓ $f(x) = 0$ ✓ factors ✓ both x values ✓ answer (4)
5.4	$f(-2) = -\frac{1}{2}(-2)^2 + \frac{1}{2}(-2) + 1$ $= -2$ PR = 2 units	✓ substitution of -2 ✓ answer (2)
5.5.1	$h(x) = q^x$ $\frac{1}{4} = q^2$ $q = \frac{1}{2}$	✓ $A\left(2; \frac{1}{4}\right)$ ✓ substitution of $\left(2; \frac{1}{4}\right)$ ✓ value of q (3)
5.5.2	$h(x) = \left(\frac{1}{2}\right)^x$ $h^{-1}(x) = \log_{\frac{1}{2}} x$	✓ equation of h ✓ equation of h^{-1} (2)
5.6	$y > 0$	✓ answer (1)
5.7	$f'(x) = -x + \frac{1}{2}$ $f'(-1) = -(-1) + \frac{1}{2} = \frac{3}{2}$ $y - y_1 = m(x - x_1)$ B(-1; 0) $y - 0 = \frac{3}{2}(x - (-1))$ $y = \frac{3}{2}x + \frac{3}{2}$	✓ derivative ✓ derivative $x = -1$ ✓ $f'(-1) = \frac{3}{2}$ ✓ substitution of B ✓ tangent (5)
5.8	$x < -1 \text{ or } x > 2$ OR $x \in (-\infty; 1) \cup (2; \infty)$	✓✓ answer (2)
		[21]

QUESTION 6		
6.1	$A = P(1+i)^n$ $2x = x(1+0,054)^n$ $2 = (1,054)^n$ $\log(2) = n \log(1,054)$ $n = \frac{\log 2}{\log(1,054)}$ $n = 13,18$	✓ substitution into correct formula ✓ correct use of logs ✓ answer (3)
6.2	$P = \frac{x \left[1 - (1+i)^{-n} \right]}{i}$ $P = \frac{3300 \left[1 - \left(1 + \frac{0,12}{12} \right)^{-60} \right]}{\frac{0,12}{12}}$ $= R148351,63$	✓ value of i ✓ value of n ✓ substitution into correct formula ✓ answer (4)
6.3	$i_{eff} = \left(1 + \frac{0,05}{4} \right)^4 - 1$ $= 0,05094533691$ $\text{Rate} = 5,09\%$	✓ value of n ✓ substitution into correct formula ✓ rate (3)
6.4	$10\,000\,000 = \frac{x \left[\left(1 + \frac{0,15}{12} \right)^{384} - 1 \right]}{\frac{0,15}{12}}$ $125000 = x(116,9484518)$ $x = R1068,85$	✓ value of n ✓ substitution into correct formula ✓ answer (3)
		[13]

QUESTION 7	Penalty for notation error in 7.1 only.	
7.1	$f(x) = -2x^2 + 1$ $f(x+h) = -2(x+h)^2 + 1$ $f(x+h) = -2x^2 - 4xh - 2h^2 + 1$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h)^2 - f(x)}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{-2x^2 - 4xh - 2h^2 + 1 - (-2x^2 + 1)}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{-2x^2 - 4xh - 2h^2 + 1 + 2x^2 - 1}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{-4xh - 2h^2}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{h(-4x - 2h)}{h}$ $= -4x$	$\checkmark -2x^2 - 4xh - 2h^2 + 1$ $\checkmark \text{ correct substitution into formula}$ $\checkmark \text{ simplification}$ $\checkmark \text{ common factor}$ $\checkmark \text{ answer} \quad (5)$
7.2.1	$f(x) = \frac{1}{2}x^2 - 5x^{-1}$ $f'(x) = x + 5x^{-2}$	$\checkmark -5x^{-1}$ $\checkmark x$ $\checkmark 5x^{-2} \quad (3)$
7.2.2	$D_x \left[\frac{-2x^2 + \sqrt[4]{x}}{x^2} \right]$ $= D_x \left[-\frac{2x^2}{x^2} + \frac{x^{\frac{1}{4}}}{x^2} \right]$ $= D_x \left[-2 + x^{-\frac{7}{4}} \right]$ $= -\frac{7}{4}x^{-\frac{11}{4}}$	$\checkmark \sqrt[4]{x} = x^{\frac{1}{4}}$ $\checkmark -2 + x^{-\frac{7}{4}}$ $\checkmark \text{ derivative} \quad (3)$
		[11]

QUESTION 8							
8.1		1	−1	5	8	−12	✓ quadratic factor ✓ factors ✓ x-values ✓ answer (4)
			0	−1	4	12	
			−1	4	12	0	
	$(x - 1)(-x^2 + 4x + 12) = 0$ $(x - 1)(x^2 - 4x - 12) = 0$ $(x - 1)(x - 6)(x + 2) = 0$ $x = 1 \text{ or } x = 6 \text{ or } x = -2$ $\therefore C(6 ; 0)$						
8.2	$-3x^2 + 10x + 8 = 0$ $3x^2 - 10x - 8 = 0$ $(3x + 2)(x - 4) = 0$ $x = -\frac{2}{3} \text{ or } x = 4$ $f(4) = -(4)^3 + 5(4)^2 + 8(4) - 12$ $= 36$						✓ derivative ✓ $f'(x) = 0$ ✓ factors ✓ x-values ✓ answer (5)
8.3	$-6x + 10 > 0$ $x < \frac{5}{3}$						✓ inequality ✓ answer (2)
8.4	$f'(x) = -3x^2 + 10x + 8$ $f'(0) = 8$ $-3x^2 + 10x + 8 = 8$ $-3x^2 + 10x = 0$ $x(-3x + 10) = 0$ $x = 0 \text{ or } x = \frac{10}{3}$ $\therefore \text{OG} = \frac{10}{3}$						✓ $f'(0) = 8$ ✓ equating ✓ factors ✓ x-values ✓ value of OG (5)
8.5	$f'(x) = -3x^2 + 10x + 8$ $f'(5) = -3(5)^2 + 10(5) + 8$ $= -17$ $m = \frac{1}{17}$						✓ substitution ✓ $f'(5) = -17$ ✓ answer (3)
							[19]

QUESTION 9		
9.1	$m_{mn} = -\frac{b}{a}$ $y = -\frac{b}{a}x + b$	✓ m_{mn} ✓ answer (2)
9.2	Area of PTOR = TP x PR $= x \left(-\frac{b}{a}x + b \right)$ $= -\frac{b}{a}x^2 + bx$ Max: $-\frac{2b}{a}x + b = 0$ $x = b \left(\frac{a}{2b} \right)$ $x = \frac{a}{2}$ $y = -\frac{b}{a} \left(\frac{a}{2} \right) + b = \frac{b}{2}$ Midpoint of MN: $\left(\frac{0+a}{2}; \frac{0+b}{2} \right) = \left(\frac{a}{2}; \frac{b}{2} \right)$ ∴ Max at midpoint	✓ substitution into area formula ✓ simplification ✓ derivative = 0 ✓ value of x ✓ value of y ✓ midpoint (6)
		[8]

QUESTION 10		
10.1.1	$45 - 38 = 7$	✓ answer (1)
10.1.2	$S = 45$ 	✓ 8 ✓ 6 ✓ $11 - x$ ✓ $7 - x$ (4)
10.1.3	$3 + 8 + 6 + 5 + 11 - x + x + 7 - x + 7 = 45$ $x = 2$	✓ calculation ✓ answer (2)
10.1.4	$P(2\text{languages}) = \frac{6 + 8 + 2}{45}$ $= \frac{16}{45}$	✓ 16 ✓ $\frac{16}{45}$ (2)
10.2.1	$8! = 40320$	✓ 8! (1)
10.2.2	$3! \times 5! \times 6 = 4320$ OR $3! \times 6! = 4320$	✓ 3! ✓ 5! ✓ $3! \times 5! \times 6$ (3)
10.2.3	$\frac{5!}{8!} = \frac{1}{336}$	✓ 5! ✓ $\frac{5!}{8!}$ OR $\frac{1}{336}$ (2)
		[15]

TOTAL: 150